On another proof of the formula E = m.c² according to the rotary theory

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Abstract. *The paper is dedicated to one of the greatest breakthroughs in the classical physics at the beginning of the 20-th century – the appearance of the special theory of relativity of Albert Einstein in 1905. In it, by the help of the rotary theory, a new proof of the most famous formula in the world – the equation giving the connection between the energy and the mass of the bodies, is presented. Rotary theory appeared in 1998, trying to explain the electromagnetic phenomena from another point of view and to answer to series of questions connected with the basic electromagnetic laws, reaching the same results but giving simpler and direct answers compared with the classical electromagnetic theory of Maxwell. In it, by the help of the method of moments, the vector of the magnetic field intensity $\vec{H}$ and the vector of the magnetic flux density $\vec{B}$ are presented as moments of the vector of the current density of the tangential displacement current $\vec{j}_{D_\tau}$, claiming in this way that the magnetic field is a form of rotating electric field. The result is a set of electromagnetic equations in electrical form, depicting all the electromagnetic phenomena from another point of view.*

Keywords: energy and mass of the bodies, special theory of relativity, Maxwell's electromagnetic theory, rotary theory of the electromagnetic field

1   Introduction

The set of Maxwell's equations appeared in the period between 1861 and 1873 (Maxwell, 1861, 1865, 1873). In 1890 they were corrected by Hertz for moving objects with speeds smaller than the speed of light (Hertz, 1890). Today they are well known as Maxwell-Hertz equations. In 1904 Lorentz made an attempt to present them for moving media (Lorentz, 1904). In 1905, in his first paper on special theory of relativity (STR), Einstein made the next complement to these equations for two different cases – for displacement currents and for displacement and convection currents (Einstein, 1905). In 1908 Einstein and Laub presented the same equations in more concise form (Einstein, 1908). Today that set is known as Maxwell-Hertz-Einstein system of basic equations of the electromagnetic field. It is concerned with moving objects with arbitrary velocities less than the speed of light *c* or even equal to it. In (Einstein, 1905) in paragraph 10 "Dynamics of the Slowly Accelerated Electron" Einstein extracted the formula for the kinetic energy *E* of the electron:

$$E = m.c^2 = \frac{m_0.c^2}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma.m_0.c^2 \tag{1}$$

where m_0 is its rest mass, *v* is its velocity, *c* is the speed of light and *γ* is the coefficient of relativity, i.e.

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{2}$$

And that extraction was done on the base of the connection between the momentum and the energy of a moving object (Feynmann, 1964a, 1964b), (Kittel, 1963), (Pauli, 1983), Savelev, 1977). In (Feynmann, 1964b, p. 28-4) Feynmann claims that Einstein was not the only one who had extracted such a formula. The reason is hidden in the classical electromagnetic theory of Maxwell, which gives the possibility to find the momentum and the energy of the electromagnetic field, from which connection that formula can be extracted.

In 1998 the rotary theory (RT) appeared trying to explain the electromagnetic phenomena from another point of view connected with the relative rotation of the electric field of the moving charges towards a static observer (Panov, 2015, 2016). In that way RT could answer to series of questions connected with the basic electromagnetic laws, reaching the same results but giving simpler and direct answers starting from the basic terms of the basic magnetic quantities – the vector of the magnetic field intensity $\vec{H}$, the vector of the magnetic flux density $\vec{B}$ and some others, reaching a new model of propagation of the electromagnetic wave in free space. RT is based on 10 physical theorems. It introduces 2 new principles and leads to rearrangement of the Maxwell's set of equations, showing that it contains equations similar to the current and voltage equations according to the Kirchhoff's laws of the electric circuits.

Today, Maxwell-Hertz-Einstein system of basic equations of the electromagnetic field is a substantial part of many basic scientific sources giving the bases of the modern electromagnetic field theory (Feynmann, 1964a), (Purcell, 1965), (Smythe, 1989), (Simonyi, 1966), (Meerovich, 1966). The last researches on the base of RT gave an additional impact over the development of that very important branch of the modern science. That is because in (Panov, 2017a) a complement (correction) to Maxwell-Hertz-Einstein system of basic equations of the electromagnetic field was done by the help of RT, extracting the relative relations towards the vector of the current density of the tangential displacement current $\vec{j}_{D_\tau}$ and the effective radius-vector $\vec{R}_{eff}$. In (Panov, 2017b), by the help of RT, a new $\vec{j}_{D_\tau}$-wave equation of the plain electromagnetic wave and the Ohm's law in differential form for the vectors $\vec{j}_{D_\tau}$ and $\vec{E}$ were introduced, being a direct mathematical proof that its propagation is directly connected with a flowing of tangential rotary displacement currents in the free space as it was declared in (Panov, 2015) few years ago. This process is connected with very complex whirling-translational flowing of displacement currents. The propagation of the electromagnetic wave in space is accompanied by the spread (movement) of electric whirls ahead in space – a phenomenon that is linked to the manifestation of complex electric processes. All these new proofs validated the existence of RT together with Maxwell's electromagnetic theory and Einstein's STR being their complement. In the present paper, a new proof of the validity of RT is presented on the base of a new proof of the most famous equation in the world as some specialists claim, i.e. that is equation (1).

2   Analysis

According to RT (Panov, 2015, 2016) the vector of the magnetic field intensity $\vec{H}_M$ and the vector of the magnetic flux density $\vec{B}_M$ at a given point M in space have the following forms:

$$\vec{H}_M = \vec{R}_{eff} \times \vec{j}_{D_\tau} = \vec{R}_{eff} \times \frac{\vec{D}_\tau}{\partial t} = \vec{R}_{eff} \times \frac{\varepsilon_0 . \varepsilon_r . \vec{E}_\tau}{\partial t} \qquad (3)$$

$$\vec{B}_M = \frac{\varepsilon_r \mu_r}{c^2}\left(\vec{R}_{eff} \times \frac{\vec{E}_\tau}{dt} \right) = \frac{\varepsilon_r \mu_r}{c^2} . \vec{M}_{\frac{\vec{E}_\tau}{dt}} \qquad (4)$$

Here, $\vec{E}_\tau$ is the tangential component of the vector of the electric field intensity, which is responsible for the existence of the magnetic field and $\frac{\vec{E}_\tau}{dt}$ is its first derivative towards time t. Except that $\vec{E}_\tau$ is a

differential quantity, and ε_r and μ_r are the relative permittivity and the relative permeability of the medium. $\vec{M}_{\frac{\vec{E}_\tau}{dt}}$ is the moment of the quantity $\frac{\vec{E}_\tau}{dt}$. And for vacuum ($\varepsilon_r = \mu_r = 1$) the following interesting relation exists:

$$\vec{M}_{\frac{\vec{E}_\tau}{dt}} = \vec{B}_M . c^2 \qquad (5)$$

which resembles equation (1). The moment $\vec{M}_{\frac{\vec{E}_\tau}{dt}}$ has an extremely large value for even small value of the vector $\vec{B}_M$. And that fact was mentioned for first time many years ago in paper (Panov, 2015)… At the same time it is possible to compare equations (1) and (4) and to receive the following relation:

$$\frac{E}{\gamma . m_0} = c^2 = \frac{\vec{M}_{\frac{\vec{E}_\tau}{dt}}}{\frac{1}{\varepsilon_r . \mu_r} . \vec{B}_M} \qquad (6)$$

which someone may say that has no practical significance, because it makes an analogy (a very far connection) between mechanics and electrodynamics. However, as we can see later the left hand side of that equation can be extracted directly from its right hand side! That is the main goal of that paper.

In order to do that let us imagine that in point M we have a moving positive charge ∂q with a uniform speed $\vec{v}_x$ along the x-axis of a static Cartesian coordinate system S. The charge has a mass m. And let us imagine a moving Cartesian coordinate system S' with a uniform speed $\vec{V} = \vec{v}_x$, in which the direction of the x'-axis coincides with the x-axis of the static coordinate system S (Fig. 1). The unit vectors in the coordinate system S are $\vec{i}$, $\vec{j}$, $\vec{k}$, and the unit vectors in the coordinate system S' are $\vec{i}'$, $\vec{j}'$, $\vec{k}'$. Except that, let us have a constant magnetic field with a magnetic field density $\vec{B}_M = \vec{B}_z$ in the static coordinate system S.
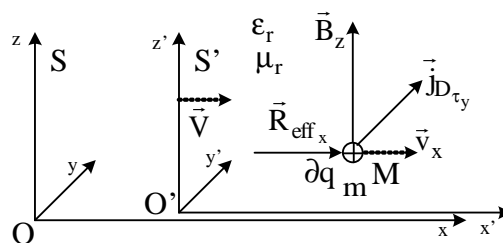

Fig. 1. A moving charge in a constant magnetic field.

The component of the magnetic flux density towards the coordinates of the moving coordinate system S' expressed by the component of the same quantity towards the coordinates of the static coordinate system S according to STR and RT are as follows (Feynmann, 1964b), (Panov, 2015), (Purcell, 1965), (Smythe, 1989), (Simonyi, 1966), (Meerovich, 1966), (Panov, 2017a):

$$\vec{B}_{z'}' = \frac{\varepsilon_r \mu_r}{c^2} . \left(\vec{R}_{eff\,x'}' \times \frac{\vec{E}_{\tau\,y'}'}{\partial t'} \right) = \gamma . \vec{B}_z = \gamma . \frac{\varepsilon_r \mu_r}{c^2} . \left(\vec{R}_{eff_x} \times \frac{\vec{E}_{\tau_y}}{\partial t} \right) \qquad (7)$$

Except that, according to Lorentz transforms in the coordinate system S', an electric field with electric field intensity $\vec{E}^{'}_{-y'}$ will also exist, i.e.:

$$\vec{E}^{'}_{-y'} = -\gamma.v_x.B_z = \vec{v}_x \times \vec{B}^{'}_{z'} \tag{8}$$

Because of the fact that the electric charge ∂q is static towards the coordinate system S', it will experience only a Coulomb force $\vec{F}^{'}_{e_{-y'}}$, which will be:

$$\vec{F}^{'}_{e_{-y'}} = \partial q.\vec{E}^{'}_{-y'} = \partial q.\left(\vec{v}_x \times \vec{B}^{'}_{z'}\right) \tag{9}$$

In the coordinate system S it will seem that the electric charge ∂q experiences a Lorentz force $\vec{F}_{L_{-y}}$. Here, both forces ($\vec{F}^{'}_{e_{-y'}}$ and $\vec{F}_{L_{-y}}$) are directed towards the negative *y*-axis. The connection between them is given by STR:

$$\vec{F}^{'}_{e_{-y'}} = \gamma \vec{F}_{L_{-y}} = \gamma.\partial q.\left(\vec{v}_x \times \vec{B}_z\right) = \partial q.\left[\vec{v}_x \times \left(\gamma.\vec{B}_z\right)\right] = \partial q.\left(\vec{v}_x \times \vec{B}^{'}_{z'}\right) = \frac{\varepsilon_r \mu_r}{c^2}.\partial q.\left[\vec{v}_x \times \left(\vec{R}^{'}_{eff_{x'}} \times \frac{\vec{E}^{'}_{\tau_{y'}}}{\partial t'}\right)\right] \tag{10}$$

According to Newton's second principle the following equality exists:

$$\vec{F}^{'}_{e_{-y'}} = \partial q.\left(\vec{v}_x \times \vec{B}^{'}_{z'}\right) = m'.\vec{a}^{'}_{n_{-y'}} \tag{11}$$

where $\vec{a}^{'}_{n_{-y'}}$ is the vector of the normal acceleration of the moving charge ∂q, having a mass m'. It is also directed towards the negative *y*-axis.

According to STR (Panov, 2017a) the following relations among the components of the quantities and the parameters in the coordinate systems S and S' exist:

$$a^{'}_{n_{-y'}} = \gamma^2.a_{n_{-y}} \quad , \quad R^{'}_{eff_{x'}} = \gamma.R_{eff_x} \quad , \quad j^{'}_{D_{\tau_{y'}}} = j_{D_{\tau_y}} \tag{12}$$

i.e. $\dfrac{\vec{E}^{'}_{\tau_{y'}}}{\partial t'} = \dfrac{\vec{E}_{\tau_y}}{\partial t}$. Then:

$$\vec{F}^{'}_{e_{-y'}} = \gamma.\vec{F}_{L_{-y}} = \partial q.\left(\vec{v}_x \times \vec{B}^{'}_{z'}\right) = \frac{\varepsilon_r \mu_r}{c^2}.\partial q.\left[\vec{v}_x \times \left(\vec{R}^{'}_{eff_{x'}} \times \frac{\vec{E}^{'}_{\tau_{y'}}}{\partial t'}\right)\right] =$$
$$= \frac{\varepsilon_r \mu_r}{c^2}.\partial q.\left[\vec{v}_x \times \left(\gamma.\vec{R}_{eff_x} \times \frac{\vec{E}_{\tau_y}}{\partial t}\right)\right] = m'.\vec{a}^{'}_{n_{-y'}} = m'.\gamma^2.\vec{a}_{n_{-y}} \tag{13}$$

In the last equation the following product $\varepsilon_r.\vec{E}_{\tau_y}$ can be replaced by $\vec{E}_{\tau_{y_{vacuum}}}$, i.e. this is the tangential component of the electric field intensity of the rotating electric whirl, responsible for the existence of the magnetic field in point M:

$$\varepsilon_r . \vec{E}_{\tau_y} = \vec{E}_{\tau_{y_{vacuum}}} \qquad (14)$$

Except that the mass *m'* of the charge in the Cartesian coordinate system S' is the so-called rest mass m_0, i.e.:

$$m' = m_0 \qquad (15)$$

If we express in equation (13) the tangential electric field intensity $\vec{E}_{\tau_y}$ according to Coulomb law by $\vec{F}_{\tau_y}$ - the tangential Coulomb force acting in point M, then it follows that:

$$\vec{E}_{\tau_y} = \frac{\vec{F}_{\tau_y}}{\partial q} = \frac{\vec{E}_{\tau_{y_{vacuum}}}}{\varepsilon_r} = \frac{\vec{F}_{\tau_{y_{vacuum}}}}{\varepsilon_r . \partial q} \qquad (16)$$

and equation (13) becomes equal to:

$$\vec{F}'_{e-y'} = \gamma . \vec{F}_{L-y} = m_0 . \gamma^2 . \vec{a}_{n-y} = \frac{\varepsilon_r \mu_r}{c^2} . \partial q . \left[\vec{v}_x \times \left(\gamma . \vec{R}_{eff_x} \times \frac{\vec{F}_{\tau_y}}{\partial t . \partial q} \right) \right] =$$

$$= \frac{\varepsilon_r \mu_r}{c^2} . \left[\vec{v}_x \times \left(\gamma . \vec{R}_{eff_x} \times \frac{\vec{F}_{\tau_{y_{vacuum}}}}{\varepsilon_r . \partial t} \right) \right] = \gamma . \frac{\mu_r}{c^2} . \left[\frac{\vec{v}_x}{\partial t} \times \left(\vec{R}_{eff_x} \times \vec{F}_{\tau_{y_{vacuum}}} \right) \right] \qquad (17)$$

Here, the vectors $\vec{R}_{eff_x}$ and $\vec{F}_{\tau_{y_{vacuum}}}$ are perpendicular and their vector product equals the torque $\vec{M}_{\tau_{z_{vacuum}}}$:

$$\vec{M}_{\tau_{z_{vacuum}}} = \vec{R}_{eff_x} \times \vec{F}_{\tau_{y_{vacuum}}} \qquad (18)$$

Having in mind the last two equations, we can extract the following relation:

$$\vec{F}_{L-y} = \gamma . m_0 . \vec{a}_{n-y} = \frac{\mu_r}{c^2} . \left[\frac{\vec{v}_x}{\partial t} \times \left(\vec{R}_{eff_x} \times \vec{F}_{\tau_{y_{vacuum}}} \right) \right] = \frac{\mu_r}{c^2} . \left(\frac{\vec{v}_x}{\partial t} \times \vec{M}_{\tau_{z_{vacuum}}} \right) \qquad (19)$$

In the last relation the vectors $\vec{v}_x$ and $\vec{M}_{\tau_{z_{vacuum}}}$ are perpendicular and their vector product is a vector which direction is along the negative *y*-axis, i.e.:

$$\vec{v}_x \times \vec{M}_{\tau_{z_{vacuum}}} = v_x . M_{\tau_{z_{vacuum}}} . (-\vec{j}) \qquad (20)$$

The vector of the normal acceleration $\vec{a}_{n-y}$ is equal to:

$$\vec{a}_{n-y} = \frac{v_x^2}{r} . (-\vec{j}) \qquad (21)$$

where *r* is the radius of the curvature. Having in mind equations (20) and (21), the following equality can be extracted from equation (19):

$$\vec{F}_{L_{-y}} = \gamma.m_0.\frac{v_x^2}{r}.(-\vec{j}) = \frac{\mu_r}{c^2}.\left[\frac{v_x}{\partial t}.M_{\tau_{z_{vacuum}}}.(-\vec{j})\right] \tag{22}$$

i.e.

$$\gamma.m_0.c^2 = \mu_r.M_{\tau_{z_{vacuum}}}.\left(\frac{v_x}{\frac{v_x^2}{r}.\partial t}\right) \tag{23}$$

If we take into account that in material media the radius of the curvature r is:

$$r = \frac{m.v}{q.B} = \frac{m.v}{q.\mu_0\mu_r.H} \tag{24}$$

and in vacuum it is r_{vacuum}, i.e.

$$r_{vacuum} = \frac{m.v}{q.B_{vacuum}} = \frac{m.v}{q.\mu_0.H} \tag{25}$$

then it is possible to do the following substitution

$$\mu_r.r = r_{vacuum} \tag{26}$$

Except that the following relation takes place for the normal component of the velocity $v_{n_{vacuum}}$:

$$\frac{v_x^2}{r_{vacuum}}.\partial t = v_{n_{vacuum}} \tag{27}$$

From the rectangular triangle of the velocities $\vec{v}_x$, $\vec{v}_n$ and $\vec{v}$, presented in Fig. 2, the following relations exist when the angle $d\varphi$ is very small (i.e. $d\varphi \approx 0$ and in practice the triangle is with two right angles):

$$\frac{v_x}{v_n} = tg(d\varphi) \approx sin(d\varphi) \approx d\varphi \tag{28}$$
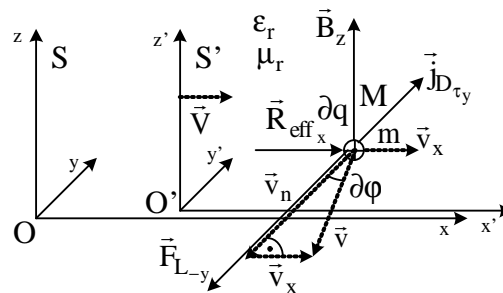
Then, equation (21) can be transformed as follows:

Fig. 2. Deflection of the moving charge in a constant magnetic field.

$$\gamma.m_0.c^2 = M_{\tau_{z_{vacuum}}}.\left(\frac{v_x}{\frac{v_x^2}{\mu_r.r}.\partial t}\right) = M_{\tau_{z_{vacuum}}}.\left(\frac{v_x}{\frac{v_x^2}{r_{vacuum}}.\partial t}\right) = M_{\tau_{z_{vacuum}}}.\left(\frac{v_x}{v_{n_{vacuum}}}\right) \qquad (29)$$

or

$$\gamma.m_0.c^2 = M_{\tau_{z_{vacuum}}}.d\varphi = W_{vacuum} \equiv E_{vacuum} = E \qquad (30)$$

Here, W_{vacuum} is the work and E_{vacuum} is the corresponding energy, which is finally denoted by E.

The final result is the most famous formula in physics (as some specialists say):

$$E = \gamma.m_0.c^2 = \frac{m_0.c^2}{\sqrt{1 - \frac{v^2}{c^2}}} = m.c^2 \qquad (31)$$

3   Conclusions

The last proof is very important for RT together with the corrected Maxwell-Hertz-Einstein system of equations of the electromagnetic field for moving media, the extraction of the new $\vec{j}_{D_\tau}$ -wave equation of the plain electromagnetic wave and the Ohm's law in differential form for the vectors $\vec{j}_{D_\tau}$ and $\vec{E}$ in it, being a direct proof that the propagation of the electromagnetic waves is directly connected with a flowing of tangential rotary displacement currents in the free space. It shows that the Maxwell's theory of the electromagnetic field, the STR of Albert Einstein and the RT are connected with very strong bonds…

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