Technical University of Varna
Annual Journal
DOI: 10.29114/ajtuv.vol2.iss1.66
Vol 2 Issue 1 (2018)
Published: 2018-06-30
ISSN: 2603-316X (Online)

Direct current regimes in the linear electric circuits according to the relativistic circuit theory

Emil Ivanov Panov[1]

[1] – Technical University of Varna, Department of Theoretical Electrical Engineering and Instrumentation, 9010, 1 Studentska Street, Varna, Bulgaria

Corresponding author contact: eipanov@yahoo.com

Abstract. *The paper is dedicated to a missing chapter of the circuit theory, which is connected with the special theory of relativity. It is concerned with the direct current regimes in the linear electric circuits, which are moving with speeds smaller than the speed of light or close to it. In it a series of basic questions, connected with the relativistic forms of the fundamental laws for the electric circuits (Kirchhoff's current law, Kirchhoff's voltage law, Ohm's law, Joule's law, the energy conservation law), are observed. The relativistic forms of the basic quantities of the electric circuits (currents, voltages, powers) and the relativistic relations of the basic parameters of the circuits (resistances, conductances, capacitances, inductances) are presented, too. These formulas are extracted step by step by the help of Maxwell-Hertz-Einstein system of basic equations of the electromagnetic field, which is applied to fast moving objects (linear electric circuits) with arbitrary velocities less than the speed of light or even close to it. The final results are illustrated by the help of some simple examples about fast moving linear electric circuits. Their analyses are presented step by step in order to show the validity of the received relations.*

Keywords: relativistic circuit theory, special theory of relativity, relativistic laws for the electric circuits, relativistic parameters, relativistic quantities

1 Introduction

The creation of the Maxwell's theory of the electromagnetic (EM) field had an enormous influence on the development of the modern physics (Maxwell, 1873). It was the main basis for the appearance of the Special Theory of Relativity (STR) of Albert Einstein (Einstein, 1905, 1908), (Feynmann, 1964a), (Kittel, 1963), which reformed the modern views on the surrounding world. One of its main consequences was the relativistic correction of the Maxwell-Hertz set of equations of the EM field for moving media (Feynmann, 1964b), (Purcell, 1965), (Simoniy, 1964) and the result was the appearance of the Maxwell-Hertz-Einstein system of basic equations of the EM field. The creation of the Rotary Theory (RT) of the EM field brought new corrections in that set of equations by the help of the method of moments (Panov, 2015, 2017a, 2017b). At the same time Circuit Theory (CT), being a consequence of the Maxwell's EM theory and its complementary theory, is not very well exposed in relativistic form, a fact which can be detected very easily in the technical literature, i.e. it is absent… The reason is connected maybe with the fact that there are some scientists, who claim directly that: …"The important consequences… (of STR) … are related to the sphere of physics, but not to electrical engineering…"… (Simonyi, 1964, p. 725). But at the same time there is another group of scientists who pay attention in their monographs to some elements of the Relativistic Circuit Theory (RCT) (which can be called also Special Circuit Theory (SCT)) (Pauli, 1958), (Meerovich, 1966), (Polivanov, 1982), (Meerovich, 1987). During the last few years some additional researches on that topic were done, but there are no generalized results towards RCT...

The main goal of that paper is to collect the existing information about the basic laws of the electric circuits in relativistic form (Kirchhoff's current law, Kirchhoff's voltage law, Ohm's law, Joule's law, the energy conservation law), the relativistic connections of the basic quantities of the circuits (currents, voltages, powers) and their basic parameters (resistances, conductances, capacitances, inductances), in order to set the basis for the analysis of direct current (DC) regimes in electric circuits, mounted in fast moving artificial objects (like satellites or space ships) with velocities less than the speed of light or close to it. Today the highest velocities reached by artificial cosmic objects are about 15 km/s and at

Technical University of Varna
Annual Journal
DOI: 10.29114/ajtuv.vol2.iss1.66
Vol 2 Issue 1 (2018)
ISSN: 2603-316X (Online)
Published: 2018-06-30

that speed the relativistic effects can be already detected… The basic problem in the research is connected with the fact that some of the elements of the explored circuits are orientated in parallel to the direction of movement, and some of them are transversely disposed, so the voltages and the currents in these moving elements can be accepted by a static observer in a different way compared with an observer, moving together with these circuits with the same speed.

2 Analysis

2.1 Basic laws of the fast moving DC linear electric circuits

According to the relativity principle of STR in each inertial coordinate system the basic forms of the laws of physics must remain the same (Einstein, 1905), (Feynmann, 1964a), (Kittel, 1963). In that relation let us imagine a moving linear electric circuit with a uniform speed $\vec{v}_x$ along the *x*-axis of a static Cartesian coordinate system *S* (Fig. 1(a)). And let us imagine a moving Cartesian coordinate system *S’* with the same uniform speed $\vec{v} = \vec{v}_x$, in which the circuit is in static position and the direction of the *x’*-axis coincides with the *x*-axis of the static coordinate system *S*.

The Maxwell’s set of equations in differential form for a static coordinating system S has the following form:

$$rot\vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t} \tag{1}$$

$$rot\vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{2}$$

$$div\vec{D} = \rho \tag{3}$$

$$div\vec{B} = 0 \tag{4}$$

The Maxwell’s set of equations in differential form for any moving coordinate system *S’* (in which the direction of the *x’*-axis coincides with the *x*-axis of the static coordinate system *S*) with a uniform speed $\vec{v}$ preserves its form (i.e. it is invariant):

$$rot\vec{H}' = \vec{j}' + \frac{\partial \vec{D}'}{\partial t'} \tag{5}$$

$$rot\vec{E}' = -\frac{\partial \vec{B}'}{\partial t'} \tag{6}$$

$$div\vec{D}' = \rho' \tag{7}$$

$$div\vec{B}' = 0 \tag{8}$$

Here, $\vec{H}$ and $\vec{H}'$ are the vectors of the magnetic field intensities, $\vec{D}$ and $\vec{D}'$ are the vectors of the electric flux densities, $\vec{E}$ and $\vec{E}'$ are the vectors of the electric field intensities, $\vec{B}$ and $\vec{B}'$ are the vectors of the magnetic flux densities, $\vec{j}'$ and $\vec{j}$ are the current densities, ρ' and ρ are the volume densities of the electric charges in both coordinate systems S' and S, correspondingly.

In more detailed form the first set of equations consists of 8 partial differential equations:

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j_x + \frac{\partial D_x}{\partial t} \tag{9}$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j_y + \frac{\partial D_y}{\partial t} \tag{10}$$

DOI: 10.29114/ajtuv.vol2.iss1.66
Vol 2 Issue 1 (2018)
Published: 2018-06-30
ISSN: 2603-316X (Online)

$$\frac{\partial H_y}{\partial x}-\frac{\partial H_x}{\partial y}=j_z+\frac{\partial D_z}{\partial t} \tag{11}$$

$$\frac{\partial D_x}{\partial x}+\frac{\partial D_y}{\partial y}+\frac{\partial D_z}{\partial z}=\rho \tag{12}$$

$$\frac{\partial E_z}{\partial y}-\frac{\partial E_y}{\partial z}=-\frac{\partial B_x}{\partial t} \tag{13}$$

$$\frac{\partial E_x}{\partial z}-\frac{\partial E_z}{\partial x}=-\frac{\partial B_y}{\partial t} \tag{14}$$

$$\frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y}=-\frac{\partial B_z}{\partial t} \tag{15}$$

$$\frac{\partial B_x}{\partial x}+\frac{\partial B_y}{\partial y}+\frac{\partial B_z}{\partial z}=0 \tag{16}$$

In more detailed form the second set of equations consists of 8 partial differential equations, too:

$$\frac{\partial H'_{z'}}{\partial y'}-\frac{\partial H'_{y'}}{\partial z'}=j'_{x'}+\frac{\partial D'_{x'}}{\partial t'} \tag{17}$$

$$\frac{\partial H'_{x'}}{\partial z'}-\frac{\partial H'_{z'}}{\partial x'}=j'_{y'}+\frac{\partial D'_{y'}}{\partial t'} \tag{18}$$

$$\frac{\partial H'_{y'}}{\partial x'}-\frac{\partial H'_{x'}}{\partial y'}=j'_{z'}+\frac{\partial D'_{z'}}{\partial t'} \tag{19}$$

$$\frac{\partial D'_{x'}}{\partial x'}+\frac{\partial D'_{y'}}{\partial y'}+\frac{\partial D'_{z'}}{\partial z'}=\rho' \tag{20}$$

$$\frac{\partial E'_{z'}}{\partial y'}-\frac{\partial E'_{y'}}{\partial z'}=-\frac{\partial B'_{x'}}{\partial t'} \tag{21}$$

$$\frac{\partial E'_{x'}}{\partial z'}-\frac{\partial E'_{z'}}{\partial x'}=-\frac{\partial B'_{y'}}{\partial t'} \tag{22}$$

$$\frac{\partial E'_{y'}}{\partial x'}-\frac{\partial E'_{x'}}{\partial y'}=-\frac{\partial B'_{z'}}{\partial t'} \tag{23}$$

$$\frac{\partial B'_{x'}}{\partial x'}+\frac{\partial B'_{y'}}{\partial y'}+\frac{\partial B'_{z'}}{\partial z'}=0 \tag{24}$$

The components of the EM quantities of the EM field in the moving coordinate system *S'*, expressed by the components of the same quantities in the static coordinate system *S* according to STR, are as follows (Meerovich, 1966, 1987):

$$E'_{x'}=E_x \tag{25}$$

$$E'_{y'}=\gamma\left(E_y-v.B_z\right) \tag{26}$$

$$E'_{z'}=\gamma\left(E_z+v.B_y\right) \tag{27}$$

$$D'_{x'}=D_x \tag{28}$$

DOI: 10.29114/ajtuv.vol2.iss1.66
Vol 2 Issue 1 (2018)
Published: 2018-06-30
ISSN: 2603-316X (Online)

$$D'_{y'} = \gamma\left(D_y - \frac{v}{c^2}.H_z\right) \tag{29}$$

$$D'_{z'} = \gamma\left(D_z + \frac{v}{c^2}.H_y\right) \tag{30}$$

$$H'_{x'} = H_x \tag{31}$$

$$H'_{y'} = \gamma\left(H_y + v.D_z\right) \tag{32}$$

$$H'_{z'} = \gamma\left(H_z - v.D_y\right) \tag{33}$$

$$B'_{x'} = B_x \tag{34}$$

$$B'_{y'} = \gamma\left(B_y + \frac{v}{c^2}.E_z\right) \tag{35}$$

$$B'_{z'} = \gamma\left(B_z - \frac{v}{c^2}.E_y\right) \tag{36}$$

$$j'_{x'} = \gamma\left(j_x - \rho.v\right) \tag{37}$$

$$j'_{y'} = j_y \tag{38}$$

$$j'_{z'} = j_z \tag{39}$$

$$\rho' = \gamma\left(\rho - \frac{v}{c_2}.j_x\right) \tag{40}$$

Here

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{41}$$

is the coefficient of relativity and c is the speed of light in vacuum, where $0 \le v \le c$. Therefore, these formulas express the Lorentz transformations of the components of the EM quantities.

The same results may be presented by the transverse and the longitudinal components of the EM quantities according to the notations of Einstein-Laub as follows (Einstein, 1908):

$$\vec{E}'_{II} = \vec{E}_{II} \tag{42}$$

$$\vec{E}'_{\perp} = \gamma\left(\vec{E}_{\perp} + \vec{v} \times \vec{B}_{\perp}\right) \tag{43}$$

$$\vec{D}'_{II} = \vec{D}_{II} \tag{44}$$

$$\vec{D}'_{\perp} = \gamma\left(\vec{D}_{\perp} + \frac{\vec{v} \times \vec{H}_{\perp}}{c^2}\right) \tag{45}$$

$$\vec{H}'_{II} = \vec{H}_{II} \tag{46}$$

$$\vec{H}'_{\perp} = \gamma\left(\vec{H}_{\perp} - \vec{v} \times \vec{D}_{\perp}\right) \tag{47}$$

$$\vec{B}'_{II} = \vec{B}_{II} \tag{48}$$

$$\vec{B}'_{\perp} = \gamma\left(\vec{B}_{\perp} - \frac{\vec{v} \times \vec{E}_{\perp}}{c^2}\right) \tag{49}$$

$$\vec{j}'_{II} = \gamma\left(\vec{j}_{II} - \rho.\vec{v}\right) \tag{50}$$

$$\vec{j}'_{\perp} = \vec{j}_{\perp} \tag{51}$$

DOI: 10.29114/ajtuv.vol2.iss1.66
Vol 2 Issue 1 (2018)
Published: 2018-06-30
ISSN: 2603-316X (Online)

$$\rho' = \gamma\left(\rho - \frac{v}{c^2}.j_{II}\right) \tag{52}$$

If we substitute equation (40) into equation (37), having in mind that $\rho' = 0$, we can receive the following result:

$$j'_{x'} = \gamma(j_x - v.\rho) = \gamma\left[j_x - v.\left(\frac{v.j_x}{c^2}\right)\right] = \frac{j_x}{\gamma} \tag{53}$$

If:

$$j'_{x'} = j'_{x'_{cond.}} \tag{54}$$

is a conduction current density,

$$j_x = j_{x cond.} + j_{x conv.} \tag{55}$$

where

$$j_{x conv.} = \frac{v^2.j_x}{c^2} \tag{56}$$

is the convection current density, i.e.

$$j'_{x'} = j'_{x'_{cond.}} = \gamma\left(j_{x_{cond.}}\right) = \frac{j_x}{\gamma} \tag{57}$$

If we accept the notations of Einstein and Laub (Einstein, 1908) for the transverse and the longitudinal components of the current densities, the current densities from equations (37) – (39) and (53) – (57) can be notated as follows:

$$j'_{x'} = j'_{x'_{cond.}} = j'_{II cond.} = j'_{II} = \gamma\left(j_{II cond.}\right) = \frac{j_{II}}{\gamma} \tag{58}$$

$$j'_{y'} = j'_{y'_{cond.}} = j'_{\perp cond.} = j'_{\perp} = j_{y_{cond.}} = j_{\perp cond.} = j_{\perp} \tag{59}$$

$$j'_{z'} = j'_{z'_{cond.}} = j'_{\perp cond.} = j'_{\perp} = j_{z_{cond.}} = j_{\perp cond.} = j_{\perp} \tag{60}$$

And if we take into account the transverse and the longitudinal cross-section areas $S'_{cond.}$ and $S_{cond.}$ ($S'_{II cond.}$, $S_{II cond.}$, $S'_{\perp cond.}$ and $S_{\perp cond.}$) of the conductors of the circuits in the coordinate systems S' and S, then we can extract the relations of the flowing currents there:

$$i'_{II cond.} = j'_{II cond.}.S'_{\perp cond.} = \gamma.j_{II cond.}.S_{\perp cond.} = \frac{j_{II}.S_{\perp cond.}}{\gamma} = \gamma.i_{II cond.} = \frac{i_{II}}{\gamma} \tag{61}$$

$$i'_{\perp cond.} = j'_{\perp cond.}.S'_{II cond.} = j_{\perp cond.}.\gamma.S_{II cond.} = j_{\perp}.\gamma.S_{II cond.} = \gamma.i_{\perp cond.} = \gamma.i_{\perp} \tag{62}$$

Using the last two equations we can formulate the Kirchhoff's current law in relativistic form. In the moving coordinate system S' it will look like:

$$\sum_{k=1}^{n} i'_k = 0 \tag{63}$$

or

$$\sum_{k=1}^{n} i'_{cond.k} = 0 \tag{64}$$

or

$$\sum_{k=1}^{n} i'_{II cond.k} + \sum_{k=1}^{n} i'_{\perp cond.k} = 0 \tag{65}$$

In the static coordinate system S it will look like:

$$\sum_{k=1}^{n} i_k = 0 \tag{66}$$

Technical University of Varna
Annual Journal
DOI: 10.29114/ajtuv.vol2.iss1.66
Vol 2 Issue 1 (2018)
ISSN: 2603-316X (Online)
Published: 2018-06-30

or

$$\sum_{k=1}^{n} i_{cond.k} = 0 \tag{67}$$

or

$$\sum_{k=1}^{n} i_{II\,cond.k} + \sum_{k=1}^{n} i_{\perp cond.k} = 0 \tag{68}$$

The relativistic relations among the last six equations are as follows:

$$\sum_{k=1}^{n} i_{II\,cond.k} + \sum_{k=1}^{n} i_{\perp cond.k} = \sum_{k=1}^{n} \frac{i_{II\,k}}{\gamma^2} + \sum_{k=1}^{n} i_{\perp\,k} = \sum_{k=1}^{n} \frac{i'_{IIcond.k}}{\gamma} + \sum_{k=1}^{n} \frac{i'_{\perp cond.k}}{\gamma} = 0 \tag{69}$$

The last equation presents Kirchhoff's current law in relativistic form. So, the algebraic sum of the conduction currents flowing through a node of an electric circuit in the coordinate system *S* or in the coordinate system *S'* is always equal to zero. It is not difficult to present Kirchhoff's current law in another form, in which the electromotive currents of the current sources (if there are such) are transferred on the right hand side of the equation and all the conduction currents, flowing through branches containing no current sources, are presented on the left hand side.

Using a similar procedure we can extract the Kirchhoff's voltage law in relativistic form, too. For that purpose if we substitute equation (35) into equation (27), having in mind that $B'_{y'} = 0$, we can receive the following result:

$$E'_{z'} = \gamma\left(E_z + v.B_y\right) = \gamma\left[E_z + v.\left(-\frac{v.E_z}{c^2}\right)\right] = \frac{E_z}{\gamma} \tag{70}$$

If:

$$E'_{z'} = E'_{z'_{cond.}} \tag{71}$$

is a component of the electric field intensity causing conduction currents,

$$E_z = E_{z\,cond.} + E_{z\,ind.} \tag{72}$$

where

$$E_{z\,ind.} = \frac{v^2.E_z}{c^2} \tag{73}$$

is the component of the electric field intensity, caused by the unipolar induction, then

$$E'_{z'} = E'_{z'_{cond.}} = \gamma\left(E_{z\,cond.}\right) = \frac{E_z}{\gamma} \tag{74}$$

If we apply the notations of Einstein and Laub (Einstein, 1908) for the transverse and the longitudinal components of the electric field intensities, the components of the electric field intensities in equations (25) – (27) and (70) – (74) can be notated as follows:

$$E'_{x'} = E'_{x'_{cond.}} = E'_{II_{cond.}} = E'_{II} = E_{x_{cond.}} = E_{II_{cond.}} = E_{II} \tag{75}$$

$$E'_{y'} = E'_{y'_{cond.}} = E'_{\perp cond.} = E'_{\perp} = \gamma\left(E_{\perp cond.}\right) = \frac{E_{\perp}}{\gamma} \tag{76}$$

$$E'_{z'} = E'_{z'_{cond.}} = E'_{\perp cond.} = E'_{\perp} = \gamma\left(E_{\perp cond.}\right) = \frac{E_{\perp}}{\gamma} \tag{77}$$

And if we take into account the transverse and the longitudinal lengths of the elements in the circuits ℓ' and ℓ ($\ell'_{II_{cond.}}$, $\ell_{II_{cond.}}$, $\ell'_{\perp cond.}$ and $\ell_{\perp cond.}$) in the coordinate systems S' and S, where the components of the electric field intensities act upon, then we can extract the relations of the voltage drops there:

$$u'_{II_{cond.}} = E'_{II_{cond.}}.\ell'_{II} = E'_{II}.\ell'_{II} = E_{II_{cond.}}.\gamma.\ell_{II} = E_{II}.\gamma.\ell_{II} = \gamma.u_{II_{cond.}} = \gamma.u_{II} \tag{78}$$

$$u'_{\perp cond.} = E'_{\perp cond.}.\ell'_{\perp} = E'_{\perp}.\ell'_{\perp} = \gamma.E_{\perp cond.}.\ell_{\perp} = \frac{E_{\perp}.\ell_{\perp}}{\gamma} = \gamma.u_{\perp cond.} = \frac{u_{\perp}}{\gamma} \tag{79}$$

DOI: 10.29114/ajtuv.vol2.iss1.66
Vol 2 Issue 1 (2018)
Published: 2018-06-30
ISSN: 2603-316X (Online)

Using the last two equations we can formulate the Kirchhoff's voltage law in relativistic form. In the moving coordinate system $S^{'}$ it will look like:

$$\sum_{k=1}^{n} u'_{k} = 0 \tag{80}$$

or

$$\sum_{k=1}^{n} u'_{cond.k} = 0 \tag{81}$$

or

$$\sum_{k=1}^{n} u'_{II\,cond.k} + \sum_{k=1}^{n} u'_{\perp cond.k} = 0 \tag{82}$$

In the static coordinate system S it will look like:

$$\sum_{k=1}^{n} u_{k} = 0 \tag{83}$$

or

$$\sum_{k=1}^{n} u_{cond.k} = 0 \tag{84}$$

or

$$\sum_{k=1}^{n} u_{II\,cond.k} + \sum_{k=1}^{n} u_{\perp cond.k} = 0 \tag{85}$$

The relativistic relations among the last six equations are as follows:

$$\sum_{k=1}^{n} u_{II\,cond.k} + \sum_{k=1}^{n} u_{\perp cond.k} = \sum_{k=1}^{n} u_{II\,k} + \sum_{k=1}^{n} \frac{u_{\perp k}}{\gamma^{2}} = \sum_{k=1}^{n} \frac{u'_{IIcond.k}}{\gamma} + \sum_{k=1}^{n} \frac{u'_{\perp cond.k}}{\gamma} = 0 \tag{86}$$

Here, $u'_{cond.}$ or $u_{cond.}$ is a voltage drop, which causes flowing of a conduction current, and

$$u_{\perp} = u_{\perp cond.} + u_{\perp ind.} \tag{87}$$

Except that $u_{\perp ind.}$ is a voltage drop caused by the unipolar induction, because of the relative movement of the circuit in the coordinate system $S^{'}$ towards the static observer in the coordinate system *S*. Equation (86) presents Kirchhoff's voltage law in relativistic form. So, the algebraic sum of the voltage drops in a contour of an electric circuit in the coordinate system *S* or in the coordinate system *S'* is always equal to zero. It is not difficult to present Kirchhoff's voltage law in another form, in which the electromotive forces of the voltage sources (if there are such) are transferred on the right hand side of the equation and all the voltage drops across the passive elements in the contour are presented on the left hand side.

The correctness of equations (58) - (62) and (75) - (79) giving the relativistic relations of the current densities, the currents, the electric field intensities and the voltage drops in both coordinate systems can be proved by Joule's law in point form for a small longitudinal or transverse element of a conductor with a current:

$$\begin{aligned} p^{'}_{II} &= \frac{P^{'}_{II}}{V^{'}_{II}} = \frac{u^{'}_{II\,cond.}.i^{'}_{II\,cond.}}{V^{'}_{II}} = \frac{(\gamma.u_{II\,cond.}).(\gamma.i_{II\,cond.})}{\gamma.V_{II}} = \frac{\gamma.u_{II\,cond.}.i_{II\,cond.}}{V_{II}} = \\ &= \frac{\gamma^{2}.P_{II}}{\gamma.V_{II}} = \frac{\gamma.P_{II}}{V_{II}} = j^{'}_{II\,cond.}.E^{'}_{II\,cond.} = \gamma.j_{II\,cond.}.E_{II\,cond.} = \gamma.p_{II} \end{aligned} \tag{88}$$

$$\begin{aligned} p^{'}_{\perp} &= \frac{P^{'}_{\perp}}{V^{'}_{\perp}} = \frac{u^{'}_{\perp cond.}.i^{'}_{\perp cond.}}{V^{'}_{\perp}} = \frac{(\gamma.u_{\perp cond.}).(\gamma.i_{\perp cond.})}{\gamma.V_{\perp}} = \frac{\gamma.u_{\perp cond.}.i_{\perp cond.}}{V_{\perp}} = \\ &= \frac{\gamma^{2}.P_{\perp}}{\gamma.V_{\perp}} = \frac{\gamma.P_{\perp}}{V_{\perp}} = j^{'}_{\perp cond.}.E^{'}_{\perp cond.} = j_{\perp cond.}.\gamma.E_{\perp cond.} = \gamma.p_{\perp} \end{aligned} \tag{89}$$

DOI: 10.29114/ajtuv.vol2.iss1.66
Vol 2 Issue 1 (2018)
Published: 2018-06-30
ISSN: 2603-316X (Online)

where p'_{II}, p_{II}, $p'_{\perp}$ and $p_{\perp}$ are the specific powers in small elements of the conductors of the explored circuit; P'_{II}, P_{II}, $P'_{\perp}$ and $P_{\perp}$ are the powers, caused by the conduction currents; V'_{II}, V_{II}, $V'_{\perp}$ and $V_{\perp}$ are the volumes of the conductors in the coordinate systems S' and S. The relations among the powers are presented in (Pauli, 1958).

2.2 Basic relations of the parameters of fast moving DC linear electric circuits

If we use the Ohm's law in point form, the relativistic relations among the conductivities of the conductors in the coordinate systems *S* and *S'* can be extracted (Meerovich, 1966):

$$j'_{II_{cond.}} = \sigma'_{II}.E'_{II_{cond.}} = \sigma'_{II}.E_{II_{cond.}} = \gamma.j_{II_{cond.}} = \gamma.\sigma_{II}.E_{II_{cond.}} \tag{90}$$

$$j'_{\perp_{cond.}} = \sigma'_{\perp}.E'_{\perp_{cond.}} = \sigma'_{\perp}.\gamma.E_{\perp_{cond.}} = j_{\perp_{cond.}} = \sigma_{\perp}.E_{\perp_{cond.}} \tag{91}$$

where σ'_{II}, σ_{II}, $\sigma'_{\perp}$ and $\sigma_{\perp}$ are the conductivities of the longitudinal and the transverse conductors of the explored circuits in the coordinate systems S' and S. From the last two equations the following relations occur:

$$\sigma'_{II} = \gamma.\sigma_{II} \tag{92}$$

$$\sigma'_{\perp} = \frac{\sigma_{\perp}}{\gamma} \tag{93}$$

The corresponding relations of the resistivities of the conductors are as follows:

$$\rho'_{R_{II}} = \frac{1}{\sigma'_{II}} = \frac{1}{\gamma.\sigma_{II}} = \frac{\rho_{R_{II}}}{\gamma} \tag{94}$$

$$\rho'_{R_{\perp}} = \frac{1}{\sigma'_{\perp}} = \frac{\gamma}{\sigma_{\perp}} = \gamma.\rho_{R_{\perp}} \tag{95}$$

Then, using the last two equations, the relations among the resistances and the coductances of the conductors of the explored circuits in the coordinate systems S' and S can be extracted:

$$R'_{II} = \frac{\rho'_{R_{II}}.\ell'_{II}}{S'_{\perp}} = \frac{1}{G'_{II}} = \frac{\rho_{R_{II}}.\gamma.\ell_{II}}{\gamma.S_{\perp}} = \frac{\rho_{R_{II}}.\ell_{II}}{S_{\perp}} = R_{II} = \frac{1}{G_{II}} \tag{96}$$

$$R'_{\perp} = \frac{\rho'_{R_{\perp}}.\ell'_{\perp}}{S'_{II}} = \frac{1}{G'_{\perp}} = \frac{\gamma.\rho_{R_{\perp}}.\ell_{\perp}}{\gamma.S_{II}} = \frac{\rho_{R_{\perp}}.\ell_{\perp}}{S_{II}} = R_{\perp} = \frac{1}{G_{\perp}} \tag{97}$$

Another important relations can be extracted for the capacitances (C'_{II}, C_{II}, $C'_{\perp}$, $C_{\perp}$) and the inductances (L'_{II}, L_{II}, $L'_{\perp}$, $L_{\perp}$) of the reactive elements of the explored electric circuits in the coordinate systems S' and S:

$$q'_{C'_{II}} = C'_{II}.u'_{II_{cond.}} = C'_{II}.\gamma.u_{II_{cond.}} = q_{C_{II}} = C_{II}.u_{II_{cond.}} = inv. \tag{98}$$

$$q'_{C'_{\perp}} = C'_{\perp}.u'_{\perp_{cond.}} = C'_{\perp}.\gamma.u_{\perp_{cond.}} = q_{C_{\perp}} = C_{\perp}.u_{\perp_{cond.}} = inv. \tag{99}$$

$$C'_{II} = \frac{C_{II}}{\gamma} \tag{100}$$

$$C'_{\perp} = \frac{C_{\perp}}{\gamma} \tag{101}$$

$$\Psi'_{L'_{II}} = L'_{II}.i'_{II_{cond.}} = L'_{II}.\gamma.i_{II_{cond.}} = \Psi_{L_{II}} = L_{II}.i_{II_{cond.}} = inv. \tag{102}$$

$$\Psi'_{L'_{\perp}} = L'_{\perp}.i'_{\perp_{cond.}} = L'_{\perp}.\gamma.i_{\perp_{cond.}} = \Psi_{L_{\perp}} = L_{\perp}.i_{\perp_{cond.}} = inv. \tag{103}$$

DOI: 10.29114/ajtuv.vol2.iss1.66
Vol 2 Issue 1 (2018)
Published: 2018-06-30
ISSN: 2603-316X (Online)

$$L'_{II} = \frac{L_{II}}{\gamma} \tag{104}$$

$$L'_{\perp} = \frac{L_{\perp}}{\gamma} \tag{105}$$

Here, $q'_{C'_{II}}$, $q_{C_{II}}$, $q'_{C'_{\perp}}$ and $q_{C_{\perp}}$ are the electric charges of the capacitors; $\Psi'_{L'_{II}}$, $\Psi_{L_{II}}$, $\Psi'_{L'_{\perp}}$ and $\Psi_{L_{\perp}}$ are the magnetic flux linkages of the coils in the coordinate systems S' and S.

The same results can be received if we examine the stored energy of a charged capacitor ($W'_{C_{II}}$, $W_{C_{II}}$, $W'_{C_{\perp}}$, $W_{C_{\perp}}$) and a coil with a current ($W'_{L_{II}}$, $W_{L_{II}}$, $W'_{L_{\perp}}$, $W_{L_{\perp}}$) in both coordinate systems.

$$\begin{aligned} W'_{C_{II}} &= \frac{q'_{C_{II}}.u'_{C_{II}}}{2} = \frac{C'_{II}.(u'_{C_{II}})^2}{2} = \gamma.W_{C_{II}} = \frac{q_{C_{II}}.\gamma.u_{C_{II}}}{2} = \\ &= \gamma.\frac{q_{C_{II}}.u_{C_{II}}}{2} = \frac{\left(\frac{C_{II}}{\gamma}\right).(\gamma.u_{C_{II}})^2}{2} = \gamma.\frac{C_{II}.(u_{L_{II}})^2}{2} \end{aligned} \tag{106}$$

$$\begin{aligned} W'_{C_{\perp}} &= \frac{q'_{C_{\perp}}.u'_{C_{\perp}}}{2} = \frac{C'_{\perp}.(u'_{C_{\perp}})^2}{2} = \gamma.W_{C_{\perp}} = \frac{q_{C_{\perp}}.\gamma.u_{C_{\perp}}}{2} = \\ &= \gamma.\frac{q_{C_{\perp}}.u_{C_{\perp}}}{2} = \frac{\left(\frac{C_{\perp}}{\gamma}\right).(\gamma.u_{C_{\perp}})^2}{2} = \gamma.\frac{C_{\perp}.(u_{L_{\perp}})^2}{2} \end{aligned} \tag{107}$$

$$\begin{aligned} W'_{L_{II}} &= \frac{\Psi'_{L_{II}}.i'_{L_{II}}}{2} = \frac{L'_{II}.(i'_{L_{II}})^2}{2} = \gamma.W_{L_{II}} = \frac{\Psi_{L_{II}}.\gamma.i_{L_{II}}}{2} = \\ &= \gamma.\frac{\Psi_{L_{II}}.i_{L_{II}}}{2} = \frac{\left(\frac{L_{II}}{\gamma}\right).(\gamma.i_{L_{II}})^2}{2} = \gamma.\frac{L_{II}.(i_{L_{II}})^2}{2} \end{aligned} \tag{108}$$

$$\begin{aligned} W'_{L_{\perp}} &= \frac{\Psi'_{L_{\perp}}.i'_{L_{\perp}}}{2} = \frac{L'_{\perp}.(i'_{L_{\perp}})^2}{2} = \gamma.W_{L_{\perp}} = \frac{\Psi_{L_{\perp}}.\gamma.i_{L_{\perp}}}{2} = \\ &= \gamma.\frac{\Psi_{L_{\perp}}.i_{L_{\perp}}}{2} = \frac{\left(\frac{L_{\perp}}{\gamma}\right).(\gamma.i_{L_{\perp}})^2}{2} = \gamma.\frac{L_{\perp}.(i_{L_{\perp}})^2}{2} \end{aligned} \tag{109}$$

So, the relativistic relations, which were presented by equations (100), (101), (104) and (105) can be easily extracted from the last four equations. The relativistic relations of the powers in the coordinate systems S' and S are presented in (Pauli, 1958).

2.3 Numeric examples

Example 1: Given a DC electric circuit in a Cartesian coordinate system S' which is moving with a velocity $v = 260000\,km/s$ towards a static Cartesian coordinate system S (Fig. 1). Here, the electromotive force of the voltage source in the coordinate system S' is $e'_{\perp} = 2V$ and the resistances of the resistors are $R'_{II} = R'_{\perp} = 1\Omega$. All quantities and parameters in the coordinate system S' are noted with a prime (Fig. 1(a)) and these in the coordinate system S are not (Fig. 1(b)). Find the quantities and the parameters in both coordinate systems.

Technical University of Varna
Annual Journal
DOI: 10.29114/ajtuv.vol2.iss1.66
Vol 2 Issue 1 (2018)
Published: 2018-06-30
ISSN: 2603-316X (Online)

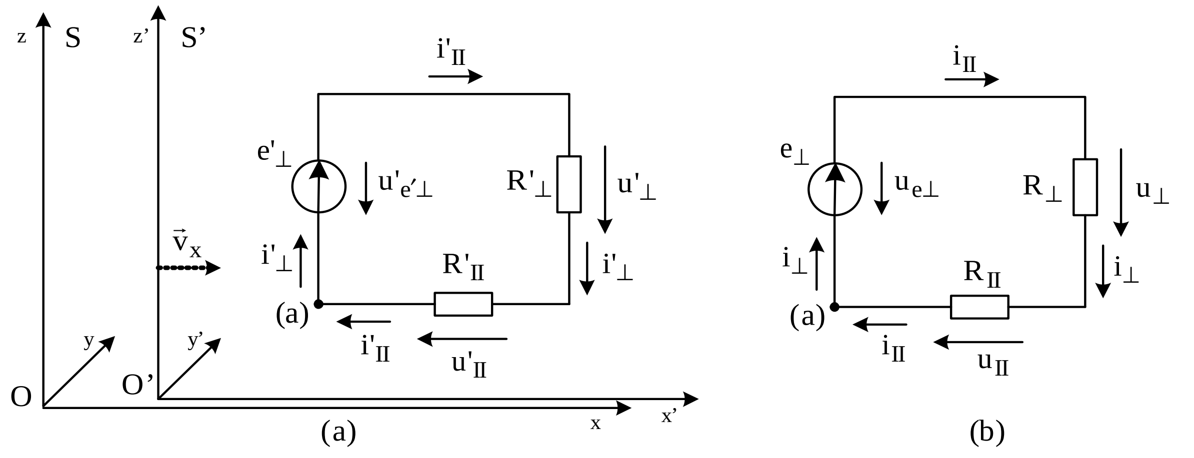

Fig. 1. A moving electric circuit with a uniform speed $\vec{v}_x$ towards a static Cartesian coordinate system S.

<u>Solution:</u>

The coefficient of relativity in this case is: $\gamma = 2$. In the coordinate system S' the following results are valid:

$$e'_{\perp} = e'_{\perp cond.} = u'_{e'_{\perp}} = u'_{e'_{\perp cond.}} = 2V = u'_{\perp cond.} + u'_{II cond.} = 1V + 1V$$

(Kirchhoff's voltage law for the loop of the circuit in Fig. 1(a));

$$i'_{II} = i'_{II cond.} = i'_{\perp} = i'_{\perp cond.} = \frac{e'_{\perp cond.}}{R'_{II} + R'_{\perp}} = \frac{2V}{1\Omega + 1\Omega} = 1A$$

(Kirchhoff's current law for node (a) in Fig. 1(a));

$$P'_{R'_{II}} = u'_{II cond.}.i'_{II cond.} = 1V.1A = 1W$$

(Power of the horizontal resistor in Fig. 1(a));

$$P'_{R'_{\perp}} = u'_{\perp cond.}.i'_{\perp cond.} = 1V.1A = 1W$$

(Power of the vertical resistor in Fig. 1(a)).
The balance of powers is as follows:

$$P'_{e'_{\perp}} = e'_{\perp cond.}.i'_{\perp cond.} = 2V.1A = P'_{R'_{II}} + P'_{R'_{\perp}} = 1W + 1W = 2W\,.$$

In the coordinate system S the following results are valid:

$$e_{\perp cond} = u_{e_{\perp cond}} = \frac{u'_{e'_{\perp cond}}}{\gamma} = \frac{2V}{2} = 1V$$

$$u_{II cond} = \frac{u'_{II cond}}{\gamma} = \frac{1V}{2} = 0{,}5V$$

$$u_{\perp cond} = \frac{u'_{\perp cond}}{\gamma} = \frac{1V}{2} = 0{,}5V$$

$$e_{\perp cond} = 1V = u_{\perp cond} + u_{II cond} = 0{,}5V + 0{,}5V = 1V$$

(Kirchhoff's voltage law for the loop of the circuit in Fig. 1(b));

$$i_{II cond.} = \frac{i'_{II cond.}}{\gamma} = \frac{1A}{2} = 0{,}5A$$

$$i_{\perp cond.} = \frac{i'_{\perp cond.}}{\gamma} = \frac{1A}{2} = 0{,}5A$$

Technical University of Varna
Annual Journal
DOI: 10.29114/ajtuv.vol2.iss1.66
Vol 2 Issue 1 (2018)
Published: 2018-06-30
ISSN: 2603-316X (Online)

$$i_{II_{cond.}} = i_{\perp_{cond.}}$$

(Kirchhoff's current law for node (a) in Fig. 1(b));

$$P_{R_{II}} = u_{II_{cond.}} . i_{II_{cond.}} = 0{,}5V . 0{,}5A = 0{,}25W$$

(Power of the horizontal resistor in Fig. 1(b));

$$P_{R_{\perp}} = u_{\perp_{cond.}} . i_{\perp_{cond.}} = 0{,}5V . 0{,}5A = 0{,}25W$$

(Power of the vertical resistor in Fig. 1(b)).
The balance of powers is as follows:

$$P_{e_{\perp}} = e_{\perp_{cond}} . i_{\perp_{cond}} = 1V . 0{,}5A = P_{R_{II}} + P_{R_{\perp}} = 0{,}25W + 0{,}25W = 0{,}5W$$

The resistances of the resistors are:

$$R_{II} = R'_{II} = 1\Omega ; \; R_{\perp} = R'_{\perp} = 1\Omega .$$

Example 2: Analyze the DC electric circuits presented in Fig. 2, which are similar to these in the previous Fig. 1, where the only difference is connected with the fact that the voltage source is disposed horizontally and its electromotive force is $e'_{II} = 2V$. The resistances of the resistors are $R'_{II} = R'_{\perp} = 1\Omega$ and the velocity of the electric circuit towards the coordinate system S is $v = 260000\,km/s$.

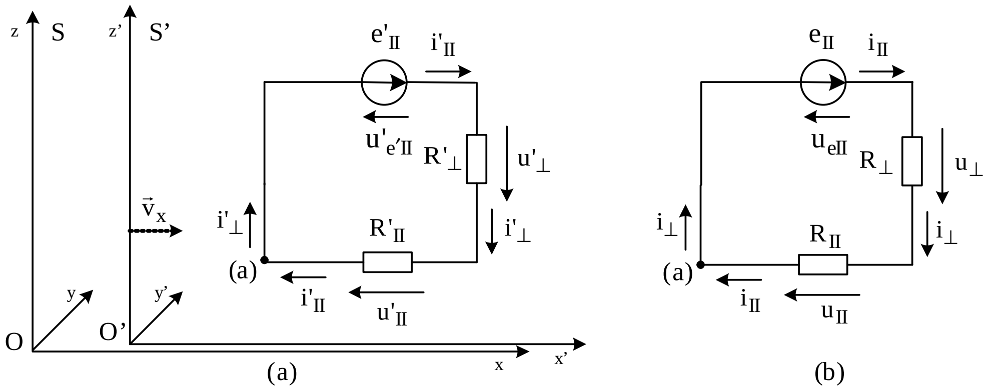

Fig. 2. A moving electric circuit with a uniform speed $\vec{v}_x$ towards a static Cartesian coordinate system S.

Solution:

The numeric results in the coordinate systems S' and S are the same as in the previous example.

The coefficient of relativity in this case is: $\gamma = 2$. In the coordinate system S' the following results are valid:

$$e'_{II} = e'_{II_{cond.}} = u'_{e'_{II}} = u'_{e'_{II_{cond.}}} = 2V = u'_{\perp_{cond.}} + u'_{II_{cond.}} = 1V + 1V$$

(Kirchhoff's voltage law for the loop of the circuit in Fig. 2(a));

$$i'_{II} = i'_{II_{cond.}} = i'_{\perp} = i'_{\perp_{cond.}} = \frac{e'_{II_{cond.}}}{R'_{II} + R'_{\perp}} = \frac{2V}{1\Omega + 1\Omega} = 1A$$

(Kirchhoff's current law for node (a) in Fig. 2(a));

$$P'_{R'_{II}} = u'_{II_{cond.}} . i'_{II_{cond.}} = 1V . 1A = 1W$$

(Power of the horizontal resistor in Fig. 2(a));

$$P'_{R'_{\perp}} = u'_{\perp_{cond.}} . i'_{\perp_{cond.}} = 1V . 1A = 1W$$

(Power of the vertical resistor in Fig. 2(a)).
The balance of powers is as follows:

$$P'_{e'_{II}} = e'_{II_{cond.}} . i'_{II_{cond.}} = 2V . 1A = P'_{R'_{II}} + P'_{R'_{\perp}} = 1W + 1W = 2W .$$

In the coordinate system S the following results are valid:

DOI: 10.29114/ajtuv.vol2.iss1.66
Vol 2 Issue 1 (2018)
Published: 2018-06-30
ISSN: 2603-316X (Online)

$$e_{II_{cond}} = u_{e_{II_{cond}}} = \frac{u'_{e_{II_{cond}}}}{\gamma} = 2V/2 = 1V$$

$$u_{II_{cond}} = \frac{u'_{II_{cond}}}{\gamma} = \frac{1V}{2} = 0{,}5V$$

$$u_{\perp cond.} = \frac{u'_{\perp cond.}}{\gamma} = \frac{1V}{2} = 0{,}5V$$

$$e_{II_{cond.}} = 1V = u_{\perp cond.} + u_{II_{cond.}} = 0{,}5V + 0{,}5V = 1V$$

(Kirchhoff's voltage law for the loop of the circuit in Fig. 2(b));

$$i_{II_{cond.}} = \frac{i'_{II_{cond.}}}{\gamma} = \frac{1A}{2} = 0{,}5A$$

$$i_{\perp cond.} = \frac{i'_{\perp cond.}}{\gamma} = \frac{1A}{2} = 0{,}5A$$

$$i_{II_{cond.}} = i_{\perp cond.}$$

(Kirchhoff's current law for node (a) in Fig. 2(b));

$$P_{R_{II}} = u_{II_{cond.}}.i_{II_{cond.}} = 0{,}5V.0{,}5A = 0{,}25W$$

(Power of the horizontal resistor in Fig. 2(b));

$$P_{R_{\perp}} = u_{\perp cond.}.i_{\perp cond.} = 0{,}5V.0{,}5A = 0{,}25W$$

(Power of the vertical resistor in Fig. 2(b)).

The balance of powers is as follows:

$$P_{e_{II}} = e_{II_{cond}}.i_{II_{cond}} = 1V.0{,}5A = P_{R_{II}} + P_{R_{\perp}} = 0{,}25W + 0{,}25W = 0{,}5W$$

The resistances of the resistors are:

$$R_{II} = R'_{II} = 1\Omega;\; R_{\perp} = R'_{\perp} = 1\Omega.$$

3 Conclusions

As a result of the research the basic laws (Kirchhoff's current law, Kirchhoff's voltage law, Ohm's law and Joule's law) in relativistic form for fast moving electric circuits working in DC regimes were extracted. A group of relativistic relations for the quantities (conduction currents, voltages, powers, electric charges and magnetic flux linkages) and the parameters (resistances, conductances, capacitances, and inductances) of these circuits are presented, too. Some relations about the powers (the balance of powers as a consequence of the energy conservation law) are shown additionally in the numeric examples.

Two numeric examples with fast moving linear electric circuits illustrate most of the extracted relations, too. All they can be very interesting to the electrical engineers who need to utilize STR in order to be able to determine the quantities and the parameters of fast moving circuits.

In this way RCT is also with open way out. Here, the explorations cover a small part of RCT, connected only with the DC regimes in linear electric circuits. The main task is to inspire the curiosity of the researchers about the fast moving electric circuits, because the relativistic effects exist around us today. In this way RCT can be very useful for prediction and explanation of new EM processes, phenomena and effects in fast moving electric circuit…

References

Einstein, A. (1905). Zur elektrodynamik bewegter körper. *Annalen der Physik*, 17, 891-921. (in German)

DOI: 10.29114/ajtuv.vol2.iss1.66
Vol 2 Issue 1 (2018)
Published: 2018-06-30
ISSN: 2603-316X (Online)

Einstein, A., Laub, J. (1908). Über die elektromagnetischen grundgleihungen für bewegte körper. *Annalen der Physik*, 331(8), 532-540. (in German)

Feynmann, R., Leighton, R., Sands, M. (1964a). *The Feynmann lectures on physics*. vol. 1, Addison Wesley Publishing House, Inc.

Feynmann, R., Leighton, R., Sands, M. (1964b). *The Feynmann lectures on physics*. vol. 2, Addison Wesley Publishing House, Inc.

Kittel, C., Knight, W. D., Ruderman M. A. (1963). Mechanics, Berkeley physics course. vol. I, N. Y., McGraw-Hill Book Co.

Maxwell, J. C. (1873). *A treatise on electricity and magnetizm*. vol. I, vol. II, Macmillan and co., London, United Kingdom.

Meerovich, E. A. (1966). *Methods of the Relativistic Electrodynamics in Electrical Engineering*. Moscow, Energia. (in Russian)

Meerovich, E. A., Meierovich, B. E. (1987). *Methods of the Relativistic Electrodynamics in Electrical Engineering and in Electrical Physics*. Moscow, Energoatomizdat. (in Russian)

Panov, E. I. (2015). *Rotary theory of the magnetic field, basics, concepts, methods and models*. (Monograph, 148 p.), LAP Lambert Academic Publishing, Germany. ISBN 978-3-659-78098-1

Panov, E. I. (2017a). On the electrodynamics of moving bodies according to the rotary theory. *Proceedings of the Second International Scientific Conference "Intelligent Information Technologies for Industry" (IITI'17), Advances in Intelligent Systems and Computing* 680, 2, Varna, Bulgaria, 280-289. Springer, (SCOPUS), ISSN 2194-5357, ISSN 2194-5365 (electronic), ISBN 978-3-319-68323-2, ISBN 978-3-319-68324-9 (eBook), doi:10.1007/978-3-319-68324-9_31

Panov, E. I. (2017b). On another proof of the formula $E = m.c^2$ according to the rotary theory, *Annual Journal of the Technical University of Varna*, 1(1), 13-20. ISSN 2603-316 (Online), doi:10.29114/adjtuv.vol1.iss1.22

Pauli, W. Jr. (1958). *Theory of relativity*. London, Pergamon Press.

Polivanov, K. M. (1982). *Electrodynamics of Moving Bodies*. Moscow, Energoizdat. (in Russian)

Purcell, E. M. (1965). *Electricity and magnetism, Berkeley physics course*. vol. II, McGraw-Hill Book Co., N. Y.

Simonyi, K. (1964). *Theoretical Electrical Engineering*. Moscow, Mir. (in Russian)